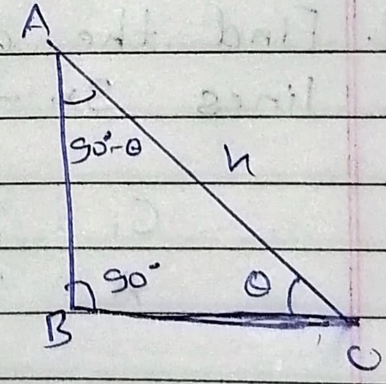
Trigonometry

Trigonometric Ratio of Angles.

Angle	0	30°	45°	60°	90°
Sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
Cosec A	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
Cot A	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Ratio of complementary Angle.

$\sin\theta = \frac{AB}{AC}$



$\cos\theta = \frac{BC}{AC}$

$\tan\theta = \frac{AB}{BC}$

$\cot\theta = \frac{BC}{AB}$, $\text{Cosec}\,\theta = \frac{AC}{AB}$, $\sec\theta = \frac{AC}{BC}$

$$\sin(90^\circ - \theta) = \frac{BC}{AC}$$

$$\cos(90^\circ - \theta) = \frac{AB}{AC}$$

$$\boxed{\frac{BC}{AC} = \frac{BC}{AC}}$$

$\sin(90^\circ - \theta) = \cos\theta$

$\cos \quad = \sin$

$\tan \quad = \cot$

$\cot \quad = \tan$

$\operatorname{cosec} \quad = \sec$

$\sec \quad = \operatorname{cosec}$

Q. Prove that $\cos 38^\circ \cos 52^\circ - \sin 38^\circ . \sin 52^\circ = 0$.

⇒ $L.H.S = \cos(90^\circ - 52^\circ) . \cos 52^\circ - \sin(90^\circ - 52^\circ) . \sin 52^\circ$

$= \sin 52^\circ . \cos 52^\circ - \cos 52^\circ . \sin 52^\circ$

$= 0$ proved

Trigonometric Identities.

$$h^2 = p^2 + b^2$$

Dividing by h^2 on both side, we have.

$$\frac{h^2}{h^2} = \frac{p^2 + b^2}{h^2}$$

$$\cancel{\frac{h^2}{h^2}} = \frac{p^2}{h^2} + \frac{b^2}{h^2}$$

$$1 = \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2$$

$$\Rightarrow \boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \sqrt{1 - \sin^2\theta}.$$

$\Rightarrow h^2 - p^2 = b^2$

$\sec^2\theta - \tan^2\theta = 1.$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec\theta = \sqrt{1 + \tan^2\theta}$

$\tan^2\theta = \sec^2\theta - 1$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

$\Rightarrow h^2 - b^2 = p^2$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\operatorname{cosec}\theta = \sqrt{1 + \cot^2\theta}$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\cot\theta = \sqrt{\operatorname{cosec}^2\theta - 1}$

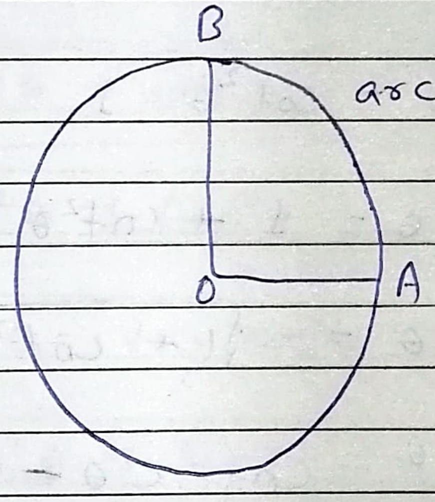
Measure of Angles

Radian or Circular measure:

Radian :

The angle subtended at the centre of circle by an arc whose length is equal to the radius of the circle is called radian.

It is denoted by 1^c.



If radius equal to 1 and arc = 1 then $\angle AOB$ is 1^c.

When θ is the angle in the radian subtended by an arc of length l at the centre of circle of radius are $\therefore$ $\boxed{\theta = \frac{l}{r}}$

Where, l = arc of the circle
r = length of radius of circle.

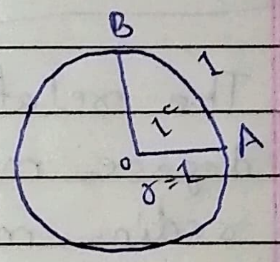
$\boxed{l = \theta r}$, $\boxed{r = \frac{l}{\theta}}$

Ex:-
Q. If arc is 8 and radius = 4 then find the radian.

$\theta = \frac{l}{r} = \frac{8}{4} = 2^c$ Ans

Relation between degree and Radian

1 Revolution = 360°.



$\Rightarrow 2\pi r = 360°$
$\Rightarrow 2\pi . 1 = 360°$

$\boxed{\pi \text{ radian} = 180°}$

$\boxed{1 \text{ radian} = \frac{180°}{\pi}}$

$$\pi^c = 180^\circ$$

$$\pi^c = 180^\circ \times 1$$

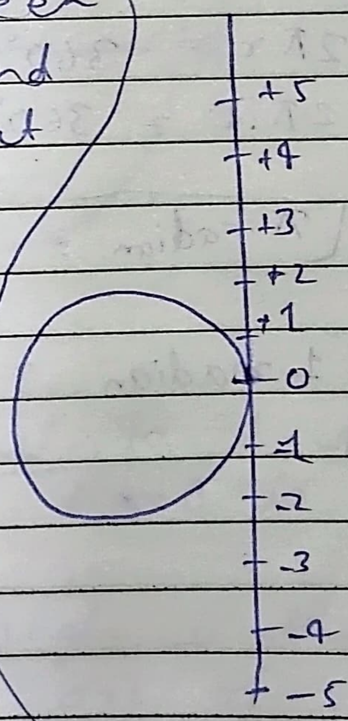
$$\cancel{\theta} = \frac{\cancel{180}\ \pi^c}{\cancel{\pi}\ 180} = 1$$

$$1^\circ = \left(\frac{\pi}{180}\right)^c$$

Relation between Radian and Real No.

→ Radian measure and Real no. are same

→ The relation between degree measure and radian measure at some common angle



Degree	0°	30°	45°	60°	90°	270°	360°
Radian	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$3\pi/2$	2π

~~Ex. 3.1~~ Exercise 3.1

1.

⇒ (i) 25°

$$1^\circ = \left(\frac{\pi}{180}\right)^c$$

$$25^\circ = \frac{\pi}{180} \times 25$$

$$\Rightarrow \quad = \frac{\pi}{\cancel{36}}$$

$$= \left(\frac{5\pi}{36}\right)^c \quad \text{Ans}$$

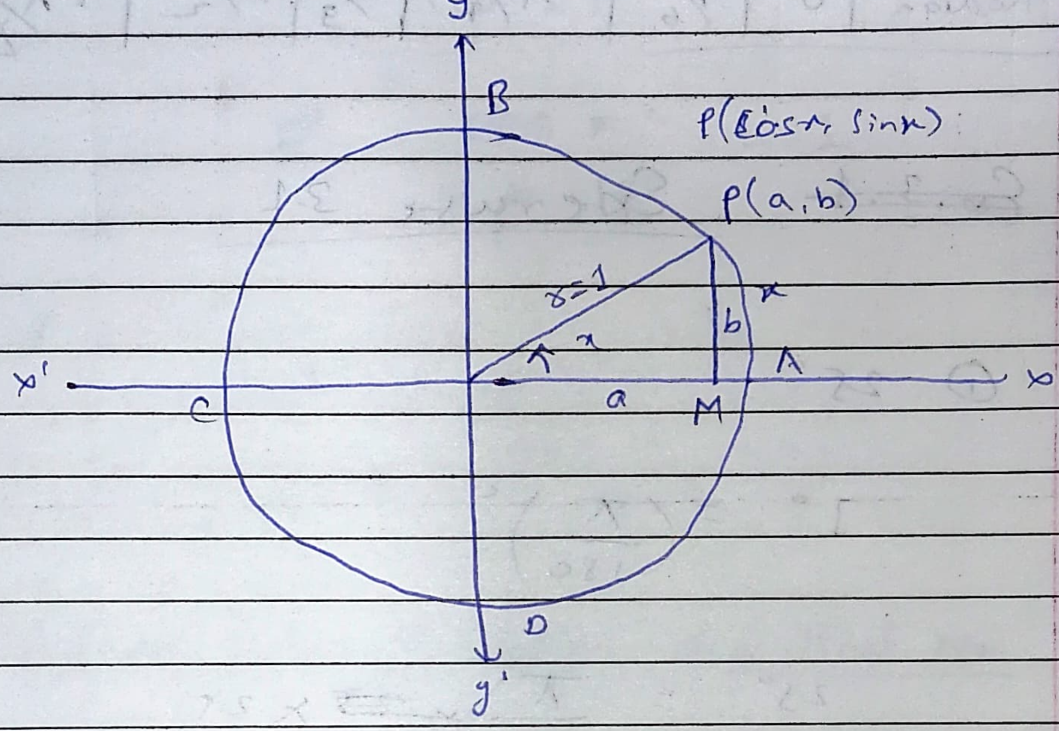
2. (i) $\dfrac{11}{16}^c$

$$1^c = \frac{180^\circ}{\pi}$$

$$\frac{11}{16} = \frac{180}{\pi} \times \frac{11}{16}$$

$$\Rightarrow \quad 37^\circ \left(\frac{8}{9} \times 60\right)'$$

Trigonometric Functions



Let P(a, b) any point on the circle with angle and arc AP $= x$

Then angle $\angle AOP = x^c$

$OM = a$

$PM = b$

$OP = 1$ (Radius)

In ΔOPM

$\sin x = \frac{P}{h}$

$\sin x = \frac{b}{1}$

$\Rightarrow b = \sin x$

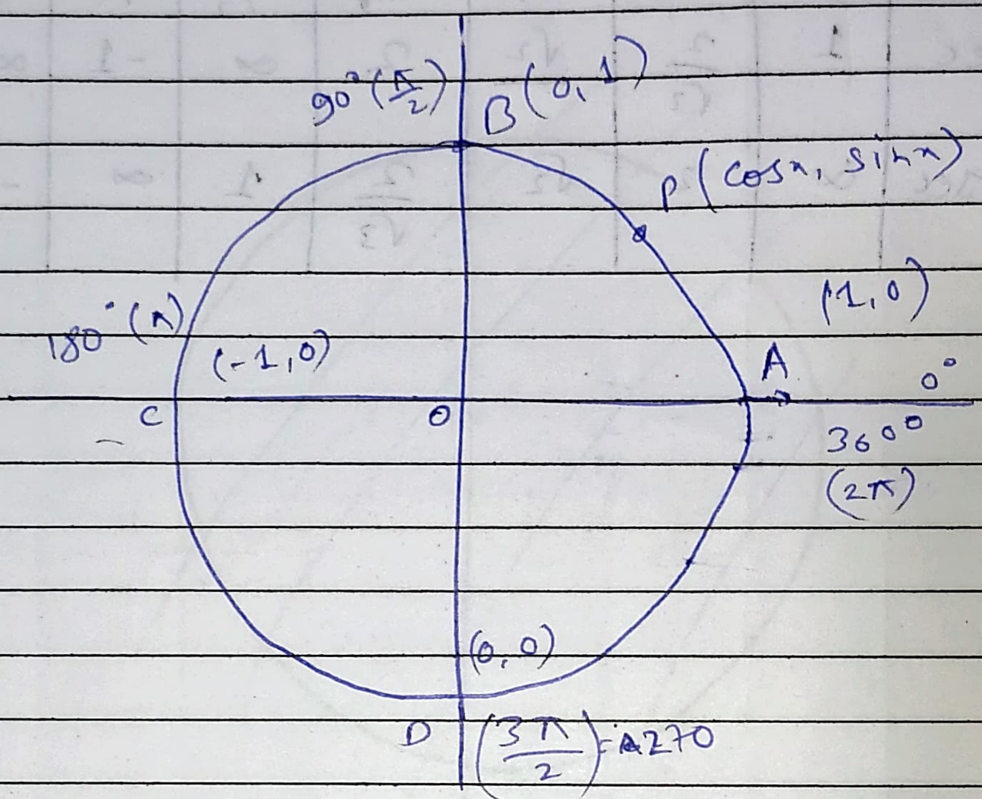
And,

$\cos x = \frac{b}{h}$

$= \frac{a}{1}$

$\Rightarrow a = \cos x.$

Therefore the point of P is $(\cos x, \sin x)$.



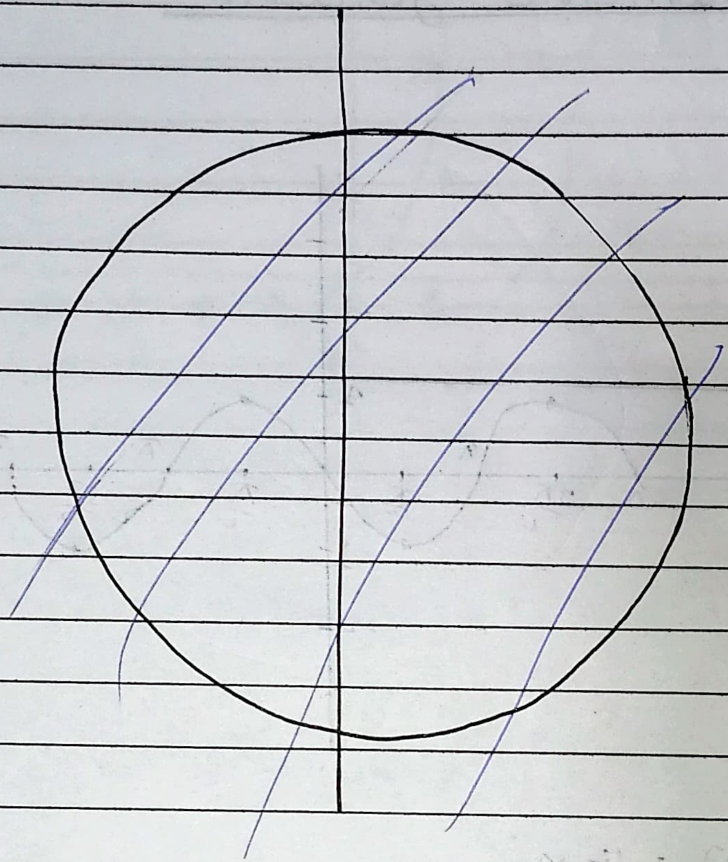
Anticlockwise

$x \rightarrow$	0°	30° $\pi/6$	45° $\pi/4$	60° $\pi/3$	90° $\pi/2$	180° π	270° $3\pi/2$	360° 2π
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞

~~Sec~~ α	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$\frac{3\pi}{2}$	2π
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	∞	1
Cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞

Co-ordinate
Matrix
Determinant
Statistics
Straight line.
Trigonometry.

Sign of Trigonometric functions



2nd quadrant

(Sin and cosec are +ve)

(Rest all -ve)

1st quadrant

(All +ve)

+ve

-ve

+ve

(tan x and cot x are +ve)

(Rest all -ve)

3rd quadrant

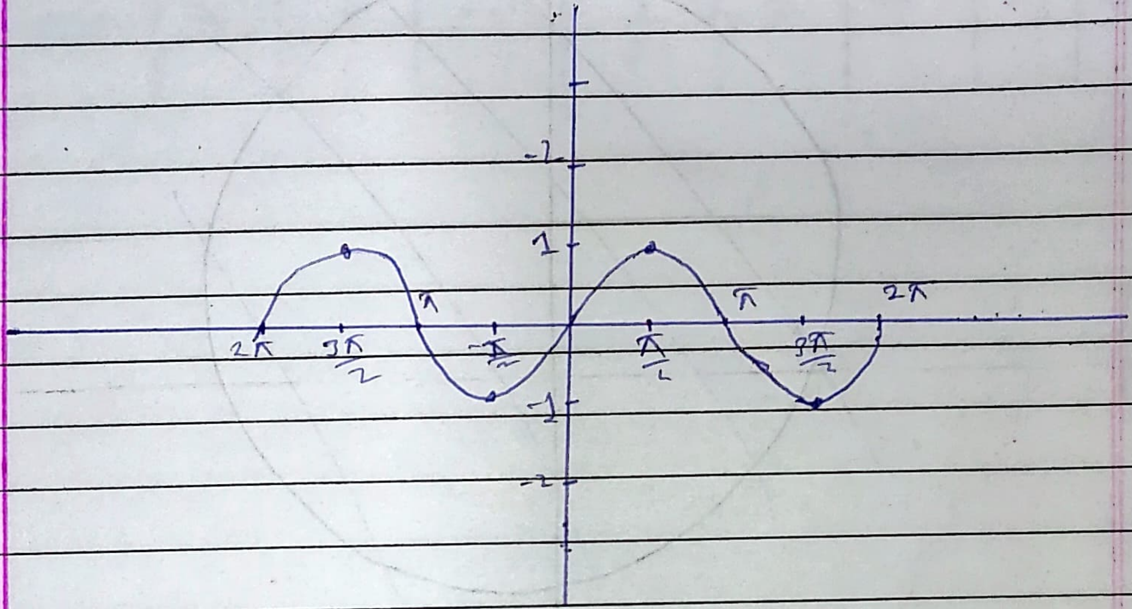
-ve

(cos x and sec x are +ve)

(Rest all -ve)

4th quadrant.

Range ~~and~~, domain and graph of trigonometric functions

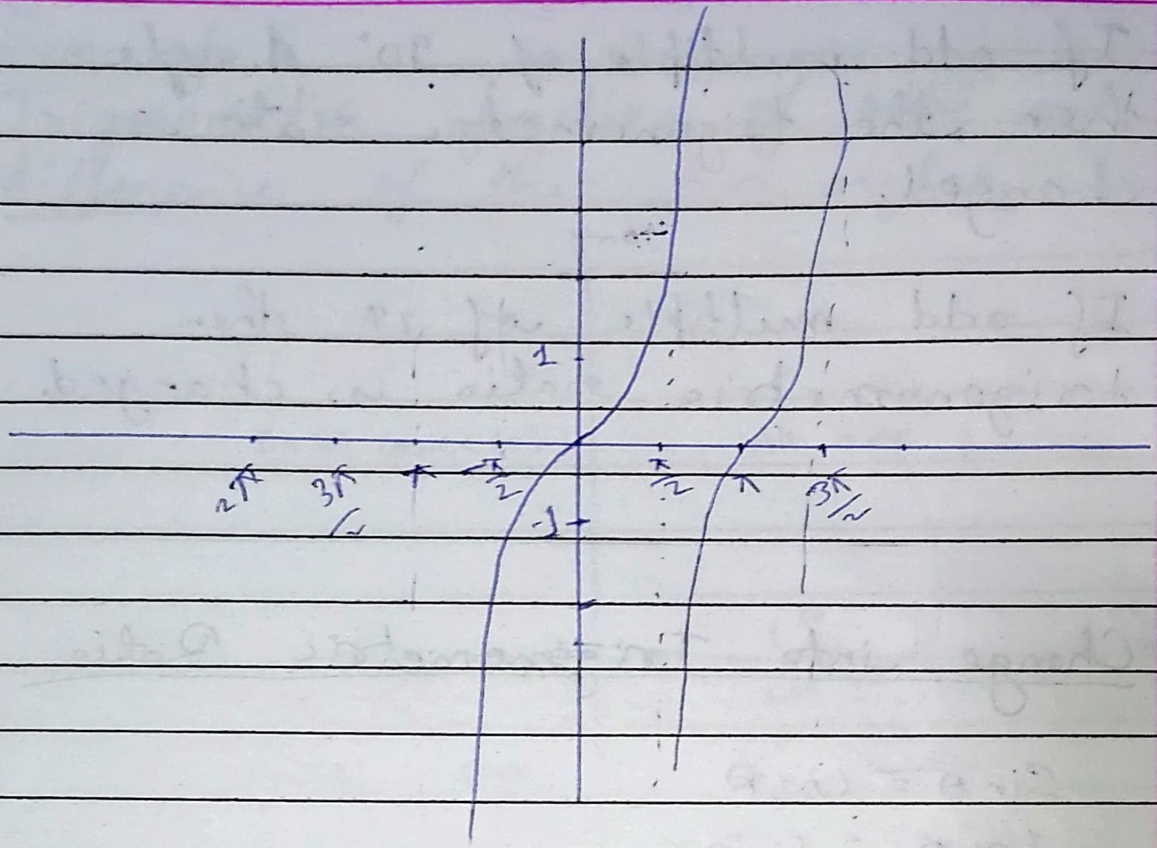


$F(x) = \sin x$

Domain $= R$

Range $= [-1, 1]$.

x



Domain ⇒ Real No. $- \left\{ (2n+1)\frac{\pi}{2} \right\}$

Range ⇒ $[-\infty, \infty]$ ~~[-1 < x < 1]~~

Note :

① If ~~any~~ ~~nx~~ even multiple of 90° angle then the trigonometric ratio is not changed.

or.

If even multiple of 2π then the trigonometric ratio is not changed.

(2) If odd multiple of 90° angle then the trigonometric ratio is changed.

or,

If odd multiple of 2π then trigonometric ratio is changed.

Change into Trigonometric Ratio

$\sin\theta = \cos\theta$

$\cos\theta = \sin\theta$

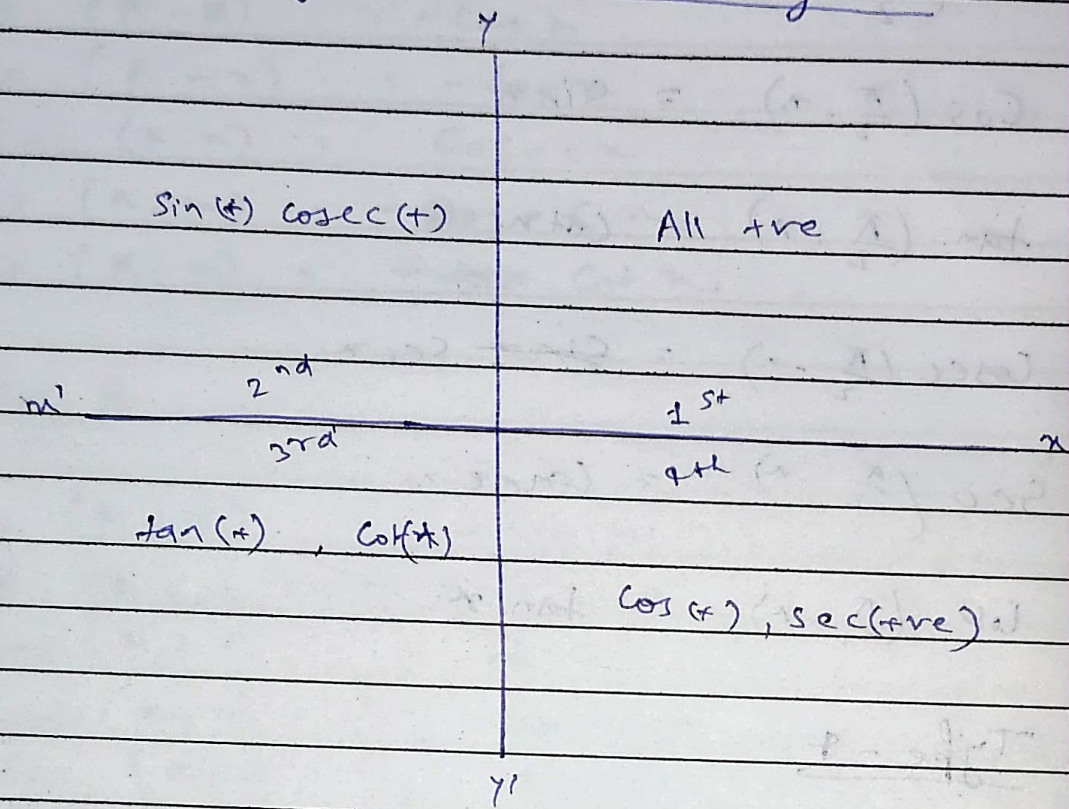
$\tan\theta = \cot\theta$

$\cot\theta = \tan\theta$

$\sec\theta = \operatorname{cosec}\theta$

$\operatorname{cosec}\theta = \sec\theta$

~~Sum of~~ Trigonometric functions of ~~some~~ sum and difference of the two angles.



Type-1

$\sin(-x) = \sin(0-x) = -\sin x.$
$\cos(-x) = \cos(0-x) = +\cos x$
$\tan(-x) = \tan(0-x) = -\tan x.$
$\cot(-x) = \cot(0-x) = -\cot x$
$\sec(-x) = \sec(0-x) = +\sec x$
$\operatorname{cosec}(-x) = \operatorname{cosec}(0-x) = -\operatorname{cosec} x.$

Type 2

$\sin(0+x) = \sin x.$
$\cos(0+x) = \cos x$
$\tan(0+x) = \cancel{\cot x}\ \tan x.$
$\operatorname{cosec}(0+x) = \operatorname{cosec} x$
$\sec(0+x) = \sec x.$
$\cot(0+x) = \cancel{\cot x}$

Type -3

$\sin\left(\frac{\pi}{2} - x\right) = \cos x.$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x.$

$\operatorname{cosec}\left(\frac{\pi}{2} - x\right) = \cancel{\sin x}\ \sec x$

$\sec\left(\frac{\pi}{2} - x\right) = \operatorname{cosec} x$

$\cot\left(\frac{\pi}{2} - x\right) = \tan x.$

Type - 4

$\sin\left(\frac{\pi}{2} + x\right) = - \cos x.$

$\cos\left(\frac{\pi}{2} + x\right) = \sin x.$

$\tan\left(\frac{\pi}{2} + x\right) = - \cot x.$

$\operatorname{cosec}\left(\frac{\pi}{2} + x\right) = - \sec x$

$\sec\left(\frac{\pi}{2} + x\right) = \operatorname{cosec} x.$

$\cot\left(\frac{\pi}{2} + x\right) = - \tan x.$

Type - 5

$\sin(\pi - x) = \sin x.$

$\cos(\pi - x) = -\cos x$

$\tan(\pi - x) = -\tan x$

$\text{cosec}(\pi - x) = \text{cosec}\, x$

$\sec(\pi - x) = -\cancel{\text{co}}\sec x$

$\cot(\pi - x) = -\cancel{\tan}\ \cot x.$

Type 6.

$\sin(\pi + x) = -\sin x.$

$\cancel{\cos(\pi}\ \cos(\pi + x) = -\cos x.$

$\tan(\pi + x) = \tan x.$

$\text{cosec}(\pi + x) = -\text{cosec}\, x$

$\sec(\pi + x) = -\sec x$

$\cot(\pi + x) = \cot x.$

Type 7.

$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

$\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$

$\tan\left(\frac{3\pi}{2} - x\right) = +\cot x$

$\text{cosec}\left(\frac{3\pi}{2} - x\right) = -\cancel{\tan x}\ \sec x$

$\sec\left(\frac{3\pi}{2} - x\right) = -\text{cosec}\, x$

$\cot\left(\frac{3\pi}{2} - x\right) = \tan x.$

Type 8.

$\sin\left(\frac{3\pi}{2}+x\right) = \cos x.$

$\cos\left(\frac{3\pi}{2}+x\right) =$

Type 9.

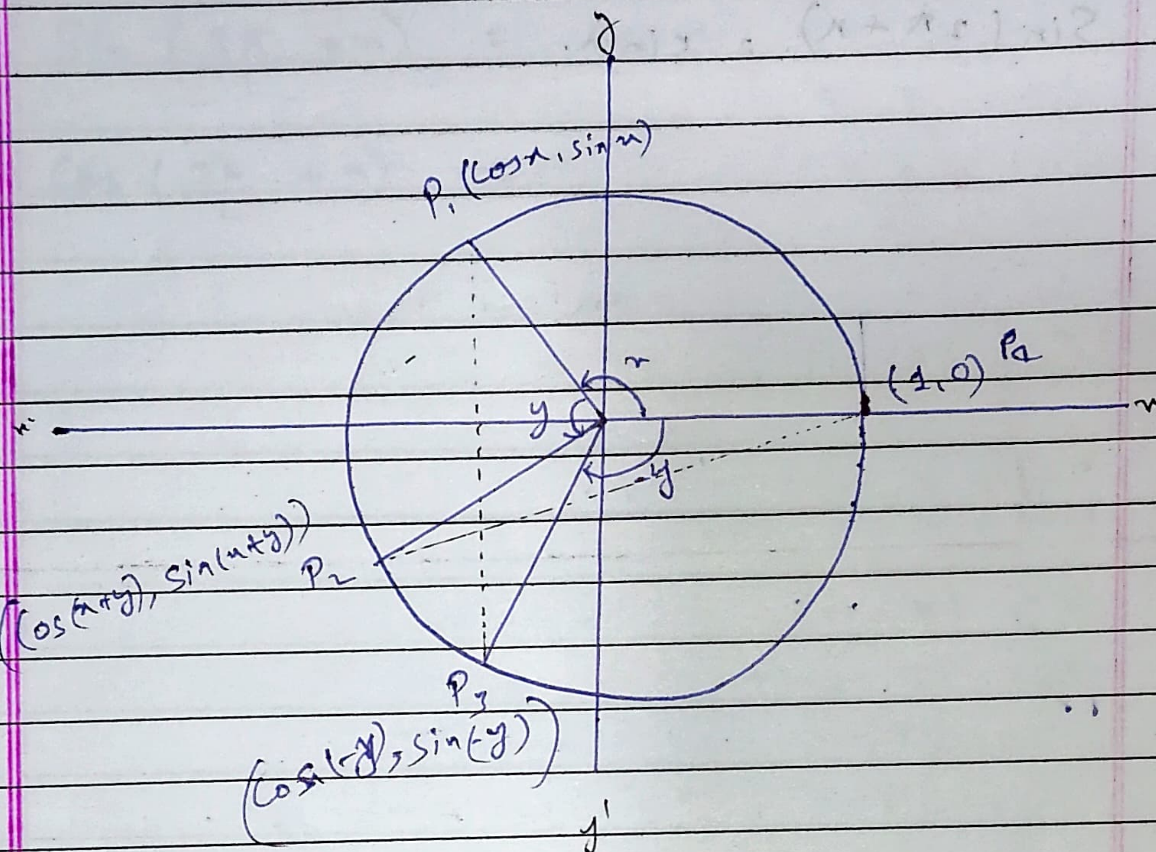
$\sin(2\pi - x) = -\sin x.$

$\cos(2\pi - x) = +\cos x.$

Type 10.

$\sin(2\pi + x) = \sin x.$

1. $\boxed{\cos(x+y) = \cos x . \cos y - \sin x . \sin y}$



Let point $P_1(\cos x, \sin y)$, $P_2[\cos(x+y), \sin(x+y)]$, $P_3[\cos(-y), \sin(-y)]$, and $P_4[1, 0]$.

Triangle $\Delta P_1 O P_3$ and $\Delta P_2 O P_4$ are congruent then $P_1 P_3$ and $P_2 P_4$ are equal.

According to distance formula

$$P_1 P_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$P_1P_3 = \sqrt{[\cos(-y) - \cos x]^2 + [\sin(-y) - \sin x)^2}$$

$$P_1P_3 = \sqrt{(\cos y - \cos x)^2 + [-\sin y - \sin x]^2}$$

Sq^2 on both sides, we have.

$$(P_1P_3)^2 = (\cos y - \cos x)^2 + (\sin y + \sin x)^2$$

$$= \cos^2 y + \cos^2 x - 2\cos x.\cos y + \sin^2 y + \sin^2 x + 2\sin x.\sin y.$$

$$(P_1P_3)^2 = 1 + 1 + 2(\sin x \sin y - \cos x.\cos y)$$

$$= 2 + 2(\sin x.\sin y - \cos x.\cos y) \quad - (1)$$

Again,

$$P_2P_4 = \sqrt{[1 - \cos(x+y)]^2 + [0 - \sin(x+y)]^2}$$

Sq^2 on both sides.

$$(P_2P_4)^2 = 1 + \cos^2(x+y) - 2\cos(x+y) + \sin^2(x+y).$$

$$= 2 - 2\cos(x+y) \quad - (ii)$$

From ① and ②, we have.

$$(P_1P_3)^2 = (P_2P_4)^2$$

$\Rightarrow 2+2(\sin x \sin y - \cos x \cos y) = 2-2\cos(x+y)$

$$\boxed{\cos(x+y) = \cos x \cos y - \sin x . \sin y}$$

Proved.

② $\boxed{\cos(x-y) = \cos x . \cos y + \sin x . \sin y}$